Third-Order Aberration Coefficients of Electron Lenses. II

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In the standard treatments of aberration coefficients of electron lenses, deviations from perfect imagery are expressed as power series of the ray coordinates in the object and aperture planes. The resulting aberration coefficients depend on the object and aperture positions, and a complete description of the aberrations of an electron lens would require a doubly infinite set of aberration coefficients for each voltage ratio of the lens. Hawkes has carried out a general treatment of the third-order aberrations of electron lenses which is independent of object and aperture positions. Six quantities are sufficient to specify the third-order aberration properties of an electron lens. We have derived equations for these six quantities in the form of integrals, involving derivatives of the axial potential no higher than the second, and using our previously calculated potentials have computed aberration coefficients for the two-tube electrostatic lens.

INTRODUCTION

In previous papers[1,2] we have pointed out the deficiencies of the standard treatments of aberration coefficients of electron lenses and the lack of data on aberration coefficients other than for spherical aberration. We adopted a method due to Verster[3] in which the third-order aberrations of electron lenses are formulated in an object- and aperture-independent way for meridional trajectories, and calculated all of Verster's aberration coefficients for the two-tube electrostatic lens for voltage ratios from 2 to 40. The method used was to fit positions and slopes of accurately calculated trajectories to the aberration equations. Because of the presence of some contribution from fifth-order aberrations, the accuracy of the third-order aberration coefficients was limited to about 10%.

To expand our treatment of aberrations to include nonmeridional (skew) trajectories we now use the more general formulation of Hawkes,[4–6] and to increase the accuracy of the aberration coefficients we calculate them from integrals involving the axial potentials and first-order trajectories. We report here results for the two-tube electrostatic lens at voltage ratios from 2 to 40.

THIRD-ORDER ABERRATION COEFFICIENTS

In the method of Verster[3] for rays in meridional planes (planes containing the lens axis), the incident and emerging rays are taken to be the asymptotes of the actual rays. The incident ray is given by its position r_1 and slope r_1' at the first focal plane, and the emerging ray by its position r_2 and slope r_2' at the second focal plane. Then to third order

$$r_2' = -(r_1/f_2) + m_{13}r_1'^3 + m_{14}r_1'^2(r_1/f_2) + m_{15}r_1'(r_1/f_2)^2 + m_{16}(r_1/f_2)^3 \quad (1)$$

and

$$r_2/f_1 = r_1' + m_{23}r_1'^3 + m_{24}r_1'^2(r_1/f_2) + m_{25}r_1'(r_1/f_2)^2 + m_{26}(r_1/f_2)^3, \quad (2)$$

where f_1 and f_2 are the focal lengths and the m_{ij} are dimensionless coefficients.

Following the more general formulation of Hawkes[4–6] let the incident asymptotic ray be defined by its direction cosines α_1, γ_1 and its coordinates x_1, y_1 at the first focal plane and the emerging asymptotic ray by its direction cosines α_2, γ_2 and its coordinates x_2, y_2 at the second focal plane. Defining the dimensionless quantities $X_1 = x_1/f_2$ and $Y_1 = y_1/f_2$, the coordinates are grouped as system invariants:

$$\begin{aligned} r &= X_1^2 + Y_1^2 \qquad s = \alpha_1^2 + \gamma_1^2 \\ u &= X_1\alpha_1 + Y_1\gamma_1 \quad v = X_1\gamma_1 - Y_1\alpha_1. \end{aligned} \quad (3)$$

The coefficient of v vanishes for electrostatic lenses.

The third-order properties are derived from a characteristic function which is second order in r, s, u:

$$(r \quad s \quad u)\begin{pmatrix} F_{11} & F_{12} & F_{13} \\ 0 & F_{22} & F_{23} \\ 0 & 0 & F_{33} \end{pmatrix}\begin{pmatrix} r \\ s \\ u \end{pmatrix} \quad (4)$$

In distinction to Hawkes' formulation, we have defined the F's as dimensionless quantities. The aberration equations are given by

$$X_2 = \alpha_1 + 4F_{11}X_1r + 2F_{12}X_1s + 2F_{13}X_1u + F_{13}\alpha_1r + (F_{23} - \tfrac{1}{2})\alpha_1s + 2F_{33}\alpha_1u \quad (5)$$

and

$$\alpha_2 = -X_1 + (F_{13} - \tfrac{1}{2})X_1r + F_{23}X_1s + 2F_{23}X_1u + 2F_{12}\alpha_1r + 4F_{22}\alpha_1s + 2F_{23}\alpha_1u, \quad (6)$$

where $X_2 = x_2/f_1$ and the corresponding equations for Y_2 and γ_2 are obtained by replacing X_1 and α_1 with Y_1 and γ_1.

TABLE I. Aberration coefficients for the two-tube lens.

V_2/V_1	F_{11}	F_{12}	F_{13}	F_{22}	F_{23}	F_{33}
2	−138.9	−197.2	−467.4	−69.85	−331.4	−393.3
5	−8.248	−7.799	−21.90	−1.760	−10.056	−14.53
10	−3.158	−2.352	−6.754	−0.3644	−2.227	−3.557
20	−1.918	−1.192	−3.077	−0.1170	−0.6862	−1.116
40	−1.631	−0.859	−1.745	−0.04882	−0.2196	−0.2680

To get Verster's equations, put $Y_1=\gamma_1=0$. Then $r=X_1^2$, $s=\alpha_1^2$, $u=X_1\alpha_1$ and we find

$$
\begin{aligned}
m_{16}&=F_{13}-\tfrac{1}{2} & m_{26}&=4F_{11}\\
m_{15}&=2F_{12}+2F_{33} & m_{25}&=3F_{13}\\
m_{14}&=3F_{23} & m_{24}&=2F_{12}+2F_{33}\\
m_{13}&=4F_{22} & m_{23}&=F_{23}-\tfrac{1}{2}.
\end{aligned}
\tag{7}
$$

CALCULATION OF COEFFICIENTS

The coefficients F were calculated from aberration integrals similar to those of Hawkes[5] which involve the axial potential distribution and its derivatives, and two independent first-order trajectories and their derivatives. For the actual calculations the original aberration integrals, involving derivatives of the axial potential as high as the fourth order, were reduced successively to integrals involving derivatives of the axial potential no higher than the third order, and then to integrals involving derivatives of the axial potential no higher than second order. Axial potentials and first-order trajectories were available from our previous calculations[7,8] of electrostatic fields and trajectories for the two-tube lens.

Aberration coefficients F were calculated for voltage ratios of 2, 5, 10, 20, and 40 using the two reduced forms of the aberration integrals. Agreement between the two forms of the integrals was typically 0.1%. The results are given in Table I.

There are no previous results with which to compare the coefficients F directly. However, using Eqs. (7) we can compute the coefficients in the Verster expansion, Eqs. (1) and (2). Agreement of these coefficients with our previously calculated values[2] is well within the estimated 10% accuracy of the previous results. Agreement with the values of Verster[3] (see Table II) determined with an automatic trajectory plotter connected to an electrolytic tank is good for m_{13}, slightly worse for m_{23} and m_{26}, but very poor for m_{16}.

We can also compare with spherical aberration coefficients for the two-tube lens calculated by Read, Adams, and Soto-Montiel[9] and by El-Kareh.[10] These coefficients, C_{s1} and C_{s2} are related to the Verster coefficients by

$$C_{s1}=-f_2m_{13} \quad \text{and} \quad C_{s2}=-f_1m_{26}.$$

Table III shows the comparison. Agreement with the accurate values of Read *et al.* is within 0.5% in almost all cases, whereas our previous results were in dis-

TABLE II. Aberration coefficients in the Verster expansion.

	V_2/V_1	2	5	10	20	40
m_{13}	Present	−279.4	−7.040	−1.458	−0.4680	−0.1953
	Previous[a]	−285	−7.34	−1.44	−0.460	−0.186
	Verster[b]		−7.32	−1.58	−0.480	−0.186
m_{14}	Present	−994.2	−30.17	−6.681	−2.059	−0.6588
	Previous	−1029	−31.6	−6.77	−2.23	−0.722
m_{15}	Present	−1181	−44.66	−11.82	−4.616	−2.254
	Previous	−1240	−47.0	−12.0	−4.27	−2.03
m_{16}	Present	−467.9	−22.40	−7.254	−3.577	−2.245
	Previous	−502	−23.6	−7.06	−3.44	−2.29
	Verster		−9.18	−1.48	−0.40	−0.23
m_{23}	Present	−331.9	−10.56	−2.727	−1.186	−0.7196
	Previous	−364	−11.0	−2.71	−1.16	−0.65
	Verster		−9.21	−3.37	−1.52	−0.90
m_{24}	Present	−1181	−44.66	−11.82	−4.616	−2.254
	Previous	−1270	−47	−11.4	−5.29	−2.19
m_{25}	Present	−1402	−65.70	−20.26	−9.231	−5.235
	Previous	−1480	−69	−19.6	−8.05	−3.03
m_{26}	Present	−555.6	−32.99	−12.63	−7.672	−6.524
	Previous	−575	−34.7	−12.2	−7.27	−6.52
	Verster		−36.8	−15.7	−9.68	−7.34

[a] Previous results are from Ref. 2.
[b] Ref. 3.

TABLE III. Comparison of spherical aberration coefficients.

	C_{s1}				C_{s2}			
V_2/V_1	Present	Previous[a]	Read *et al.*[b]	El-Kareh[c]	Present	Previous	Read *et al.*	El-Kareh
2	4343	4430	4328	4119	6107	6320	6081	5807
5	27.66	28.8	27.59	26.3	57.97	61.0	57.83	56.3
10	3.685	3.64	3.674	3.50	10.095	9.76	10.10	10.06
20	0.9512	0.936	0.9509	0.92	3.487	3.31	3.511	3.52
40	0.3689	0.352	0.3725	0.375	1.948	1.95	1.922	1.86

[a] Previous results as from Ref. 2.
[b] From Ref. 9.
[c] From Ref. 10.

agreement by as much as 5%. The poorer agreement with the results of El-Kareh is to be expected because of the use of an approximate axial potential by El-Kareh.

DISCUSSION

We have obtained, for the first time, a complete set of third-order aberration coefficients for the two-tube electrostatic lens, including skew trajectories. From these it is possible to calculate the position and slope to third order of the exit trajectory which corresponds to any incident ray. Since skew trajectories are included, it would be possible to calculate spot diagrams for electrostatic lenses, in analogy with similar calculations in light optics.[11]

From the equations of Verster[3] or of Hawkes[5] it is possible to calculate the more usual coefficients of spherical aberration, coma, astigmatism, curvature of field, and distortion for any position of the object and aperture.

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